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MAGNETIC MOMENTS OF BARYONS, G_A/G_V, AND THE QUARK CONFIGURATION STRUCTURE OF NUCLEONS

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The sum-rule-based analysis of new, precise FNAL data on hyperon magnetic moments results, in particular, in the ratio of the *D*- and *F*-type coupling constants of corresponding quark currents and baryons, which differs from the same ratio extracted from weak semileptonic decays of baryons. We consider this difference as due to a possible higher orbital configuration or exotic component admixture in baryon ground states. The amplitudes of different components of the nucleon wave function are estimated and further consequences of a general sum rule approach to baryon electroweak coupling constants are pointed out and discussed.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Магнитные моменты барионов, G_A/G_V и кварковая конфигурационная структура нуклонов

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Анализ новых прецизионных данных ФНАЛ для магнитных моментов гиперонов, выполненный на основе правил сумм, приводит к отношению констант связи *F*- и *D*-типа для соответствующих кварковых токов и барионов, которое отличается от того же отношения, определенного из данных по полулептонным распадам барионов. Мы интерпретируем это различие как обусловленное присутствием высших орбитальных кварковых конфигураций или примесью экзотической кварк-глюонной компоненты в основных состояниях барионов. Приводятся численные оценки амплитуд различных компонент в волновой функции нуклонов, а также отмечены и обсуждены дополнительные следствия применения общих правил сумм к описанию электрослабых констант связи барионов.

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1. Introduction

The rapidly growing precision of measurements of the electroweak coupling constants, like magnetic moments or the G_A/G_V — ratios in baryon semi-leptonic decays, may be the basis of new, more subtle and detailed information on the internal structure of baryons. The problem for theory is to suggest or elaborate reliable and accurate enough (i.e. adequate to the achieved experimental accuracy) tools to extract this information. In this report we consider some consequences from sum rules for the static, electroweak characteristics of baryons following from the theory of broken internal symmetries and common features of the hybrid quark models including relativistic effects and hadronic corrections due to the meson exchange currents. We choose the phenomenological sum rule techniques to obtain, at the price of a minimal number of the model-dependent assumptions, a more reliable, though not as much detailed information about the hadron properties in question.

2. The Summary of the Sum Rule Approach to $\mu(B)$

In Ref. [1,2], the following parametrization was introduced for magnetic moments $\mu(B)$ of baryons ($B = q_e^2 q_0$, $q_e = q_{even}$, $q_o = q_{odd}$, $B = P, N, \Sigma, \Xi$):

$$\mu(B) = \mu(q_e) g_2 + \mu(q_o) g_1 + C(B) + \Delta\,, \tag{1}$$

$$\mu(\Lambda) = \mu(s)\left(\frac{2}{3} g_2 - \frac{1}{3} g_1\right) + \big(\mu(u) + \mu(d)\big)\left(\frac{1}{6} g_2 + \frac{2}{3} g_1\right) + \Delta\,, \tag{2}$$

$$\mu(\Lambda\Sigma) = \frac{1}{\sqrt{3}}\big(\mu(u) - \mu(d)\big)\left(\frac{1}{2} g_2 - g_1\right) + C(\Lambda\Sigma), \tag{3}$$

$$\Delta = \sum_{q = u, d, s} \mu(q)\,\delta(N), \tag{4}$$

where $\mu(q)$ are the effective quark magnetic moments defined without any nonrelativistic approximations, $g_{2(1)}$ are «reduced» dimensionless coupling constants obeying exact $SU(3)$-symmetry, $\delta(B)$ is a matrix element of the OZI-suppressed $\bar{q}q$-configuration for a given hadron: $\delta(B) = \langle B | \bar{q}q | B \rangle$, where $q \not\subset \left\{q_e^2, q_o\right\}$, e.g., $\delta(N) = \langle N | \bar{s}s | N \rangle$, etc., $C(P) = -C(N)$ and $C(\Lambda\Sigma)$ are the isovector contributions of the charged-pion exchange current to $\mu(P)$, $\mu(N)$ and the $\Lambda\Sigma$-transition moment $\mu(\Lambda\Sigma)$. Below, we shall use the particle and quark symbols for corresponding magnetic moments. Equations (1)—(4) lead to the following sum rules [1,2].

$$P + N + \Xi^0 + \Xi^- - 3\Lambda + \frac{1}{2}\left(\Sigma^+ + \Sigma^-\right) = 0\,, \qquad (5)$$

$$\left(\Sigma^+ - \Sigma^-\right)\left(\Sigma^+ + \Sigma^- - P - N\right) - \left(\Xi^0 - \Xi^-\right)\left(\Xi^0 + \Xi^- - P - N\right) = 0, \qquad (6)$$

$$\alpha = \frac{D}{F + D} = \frac{g_2 - 2g_1}{2\,(g_2 - g_1)} = \frac{1}{2}\left(1 - \frac{\Xi^0 - \Xi^-}{\Sigma^+ - \Sigma^- - \Xi^0 + \Xi^-}\right), \qquad (7)$$

$$C\,(P) = \frac{1}{2}\left(C\,(P) - C\,(N)\right) = \frac{1}{2}\left(P - N + \Xi^0 - \Xi^- - \Sigma^+ + \Sigma^-\right), \qquad (8)$$

$$C\,(\Lambda\,\Sigma) = \mu\,(\Lambda\,\Sigma) + \frac{1}{\sqrt{3}}\left(\Xi^0 - \Xi^- - \frac{1}{2}\left(\Sigma^+ - \Sigma^-\right)\right), \qquad (9)$$

$$\frac{u - d}{u - s} = \frac{\Sigma^+ - \Sigma^- - \Xi^0 + \Xi^-}{\Sigma^+ - \Xi^0}, \qquad (10)$$

$$\frac{u}{d} = \frac{\Sigma^+\left(\Sigma^+ - \Sigma^-\right) - \Xi^0\left(\Xi^0 - \Xi^-\right)}{\Sigma^-\left(\Sigma^+ - \Sigma^-\right) - \Xi^-\left(\Xi^0 - \Xi^-\right)}, \qquad (11)$$

$$\frac{s}{d} = \frac{\Sigma^+\,\Xi^- - \Sigma^-\,\Xi^0}{\Sigma^-\,(\Sigma^+ - \Sigma^-) - \Xi^-\,(\Xi^0 - \Xi^-)}, \qquad (12)$$

$$\Delta^{++} = \frac{u}{s}\,\Omega^- = \frac{u}{d}\,\Delta^{--}. \qquad (13)$$

Reserving the possibility of $g_i\,(N) \neq g_i\,(Y)$, $Y = \Lambda, \Sigma, \Xi$, due to a more prominent role of the pion degrees of freedom in the nucleon, and combining Eqs. (5)—(6), we propose also the following, probably, most general sum rule of this approach

$$\left(\Sigma^+ - \Sigma^-\right)\left(\Sigma^+ + \Sigma^- - 6\Lambda + 2\Xi^0 + 2\Xi^-\right) -$$

$$- \left(\Xi^0 - \Xi^-\right)\left(\Sigma^+ + \Sigma^- + 6\,\Lambda - 4\Xi^0 - 4\Xi^-\right) = 0. \qquad (14)$$

We note that Eqs. (11)—(13) are obtained only with $\delta(N) = 0$, according to the quark-line-rule. Now, we list some consequences of the obtained sum rules. By definition, the Λ-value entering into Eqs. (5) and (6) should be «refined» from the electromagnetic $\Lambda\Sigma^0$-mixing affecting $\mu(\Lambda)_{\rm exp}$. Hence, the numerical value of Λ, extracted from Eq. (14), can be used to determine the $\Lambda\Sigma^0$-mixing angle through the relation

$$\sin\theta_{\Lambda\Sigma} \approx \theta_{\Lambda\Sigma} = \frac{\Lambda - \Lambda_{\rm exp}}{2\mu(\Lambda\Sigma)} = (1.43 \pm 0.31)\cdot 10^{-2} \qquad (15)$$

in accord with the independent estimate of $\theta_{\Lambda\Sigma}$ from the electromagnetic mass-splitting sum rule [3]. Equation (11) shows that owing to interaction of the u- and d-quarks with charged pions the «magnetic anomaly» is developing, i.e., $u/d = -1.80 \pm 0.02 \neq Q_u/Q_d = -2$. Evaluation of the lowest order quark-pion loop diagrams gives [2]: $u/d = \left(Q_u + \kappa_u\right)/\left(Q_d + \kappa_d\right) = -1.77$, where κ_q is the quark anomalous magnetic moment in natural units. The meson-baryon universality of the quark characteristics, Eqs. (10)—(12), suggested long ago [4], is confirmed by the calculation of the ratio of K^* radiative widths

$$\frac{\Gamma(K^{*+} \to K^+\gamma)}{\Gamma(K^{*+} \to K^+\gamma)} = \left(\frac{u/d + s/d}{1 + s/d}\right)^2 =$$

$$= 0.42 \pm 0.03 \left(vs\, 0.44 \pm 0.06 \ [5]\right). \qquad (16)$$

The experimentally interesting quantities $\mu(\Delta^+ P) = \mu(\Delta^0 N)$ and $\mu(\Sigma^{*0}\Lambda)$ are affected by the exchange current contributions and for their estimation we need additional assumptions. We use the analogy with the one-pion-exchange current, well known in nuclear physics, to assume for the exchange magnetic moment operator

$$\hat{\mu}_{exch} = \sum_{i<j} \left[\sigma_i \times \sigma_j\right]_3 \left[\tau_i \times \tau_j\right]_3 f(r_{ij}), \qquad (17)$$

where $f(r_{ij})$ is an unspecified function of the interquark distances, σ_i (τ_i) are spin (isospin) operators of quarks. Calculating the matrix elements of

$\hat{\mu}_{exch}$ between the baryon wave functions, belonging to the 56-plet of $SU(6)$, one can find

$$C(P) = \frac{1}{\sqrt{2}} C(\Delta^+ P) = \sqrt{3}\, C(\Lambda \Sigma), \qquad (18)$$

$$\mu\left(\Delta^+ P; \{56\}\right) = \frac{1}{\sqrt{2}}\left(P - N + \frac{1}{3}(P+N)\frac{1 - u/d}{1 + u/d}\right). \qquad (19)$$

We postpone further discussion and correction of Eq. (19) until our concluding section and will concentrate on confronting our $\alpha_m = 0.571 \pm 0.002$, following from Eq. (7), with corresponding values of α_{ax} from the baryon beta-decays. While evaluating Eq. (7) we used baryon magnetic moments from the PDG-tabulation [6] except $\mu(\Xi^-)$, from Ref. [7], and $\mu(\Sigma^+)_{exp} = -2.479 \pm 0.025$ n.m. from Ref. [8]. We choose the latest of two available values of $\mu(\Sigma^+)$, because it gives better agreement between α_m's calculated from the hyperon and nucleon data.

3. The $SU(3)$- and Broken-$SU(3)$-Description of Baryon Semi-Leptonic Decays

The extraction of hyperon structure parameters from their semi-leptonic decays is not yet the fully settled and understood problem. Theoretically, one should take into account the recoil and other non-static effects arising owing to differences between the initial- and final-baryon wave functions and masses. Experimentally, the $\Sigma \to \Lambda e\nu$- and $\Xi \to \Lambda e\nu$-decay measurements are not still accurate enough. We shall take as our input data the results of Refs. [9,10]. It was shown that with both $\Delta S = 1$ and 0-transition (here S is the strangeness) taken for the analysis, α_{ax} (no $SU(3)$) = $= 0.635 \pm 0{,}005$ ($\chi^2/DF = 12.5/11$) if the use is only made of the recoil correction via the recipes of Ref. [11], while the mismatch effects of the strange- and non-strange-quark wave function are neglected. Taking into account only the $\Delta S = 0$ transitions one can obtain [9] $\alpha_{ax}(SU(3)) = 0.584 \pm 0.035$ and α_{ax}(no $SU(3)$) $= 0.62 \pm 0.040$ with equal values of $\chi^2/DF = 0.07$. Therefore, we shall use the α_{ax}-values in the range $0.584 \div 0.635$.

4. The Electroweak α-Parameters and Quark Configuration Structure of the Nucleon

We consider the difference between α_m, Eq. (7), and the α_{ax}'s, given in Ref. [9,10], as originating from the higher $SU(6) \times O(3)$ three-quark configurations and/or the exotic $(3q + g)$-admixture in the ground state wave function:

$$\Psi = A_0\Psi_0(\{56\}_S, L_q = 0, S_q = 1/2) + A_1\Psi_1(\{70\}_M, L_q = 0, S_q = 1/2) +$$

$$+ A_2\Psi_2(\{70\}_M, L_q = 2, S_q = 3/2) + A_3\Psi_3(\{20\}_A, L_q = 1, S_q = 1/2) +$$

$$+ A_g\Psi_g(\{3\,q\}_{8c} + g_{8c}). \qquad (20)$$

The coefficients A_i and A_g satisfy the normalization condition

$$\sum_{i=0}^{3} A_i^2 + A_g^2 = 1. \qquad (21)$$

In Eq. (20), L_q (S_q) is the quark orbital (spin) moment, and the index «8c» stands for the color-octet states. To specify different cases, we keep for the gluon angular momentum two simplest possibilities $J_g^P = 1^{\pm}$ which are the $M1$- or $E1$-gluon modes. Different components of the total wave function are built of the antisymmetrized products of the flavor (Φ), spin (χ) color (ω) and orbital/radial (Ψ) wave functions:

$$\Psi = \Phi \times \chi \times \omega \times \Psi(\rho, \lambda), \qquad (22)$$

ρ and λ being the Jacobi coordinates of the 3-quark system. Of many considered possibilities for Ψ_g we present two examples, one for the $M1$- and one for the $E1$-gluon mode

$$\Psi_g^{M1} = \frac{1}{2}\left[\left(\Phi^{\rho}\omega^{\rho} - \Phi^{\lambda}\omega^{\lambda}\right)\chi^{\lambda} + \left(\Phi^{\rho}\omega^{\rho} + \Phi^{\lambda}\omega^{\lambda}\right)\chi^{\rho}\right]\Psi_{sym}, \qquad (23)$$

$$\Psi_g^{E1} = \frac{1}{2}\left[\left(\Psi^{\rho}\chi^{\rho} + \Phi^{\lambda}\chi^{\lambda}\right)\left(\omega^{\rho}\Psi^{\lambda} - \omega^{\lambda}\Psi^{\rho}\right)\right], \qquad (24)$$

where $\Psi_{sym}\left(L_\rho = L_\lambda = 0\right) = \Psi_0\left(\rho^2+\lambda^2\right)$, $\Psi^{\rho(\lambda)}\left(L_{\rho(\lambda)} = 1, L_{\rho(\lambda)} = 0\right) = \rho(\lambda)\cdot\Psi_1\left(\rho^2+\lambda^2\right)$, $\Psi_{0,1}\left(\rho^2+\lambda^2\right)$ are unspecified radial wave functions, $\Phi^{\rho(\lambda)}$, $\omega^{\rho(\lambda)}$ etc., are familiar, octet-type wave functions (see, e.g., Ref. [12] and earlier citations therein). Then we find expectation values of the magnetic moment $(\hat{\mu})$ and axial charge (A) operators

$$\hat{\mu} = \sum_q \left[g_\sigma(q)\,\hat{\sigma}_3(q) + g_l(q)\,\hat{l}_3(q)\right], \tag{25}$$

$$\hat{A} = \sum_q g_{ax}(q)\,\hat{\sigma}_3(q), \tag{26}$$

and define g_i^m in Eq. (1) and the analogous g_i^{ax}, as function of $A_0,\ldots,A_g$, g_σ, g_l and g_{ax}. Then we take the ratios α_m and α_{ax}; from the latter the unknown (due to various renormalization effects) g_{ax} will be cancelled out. With the known values of $\alpha_m\,\alpha_{ax}$ and Eq. (21), we have 3 equations, so that we can only define A_0^2, A_1^2 and A_2^2 as functions of A_3^2 and A_g^2, chosen to be varied over some reasonable intervals, say, $0 \le A_3^2, A_g^2 \le 0.2$., or so. Below we reproduce the corresponding bounds on the other A's, $|G_A/G_V|$ (calculated here by putting $g_{ax}(q) = 1$), and the corresponding bounds on the expectation values $<S_q>$, $<L_q>$ and $<S_g>$ entering into the angular momentum sum rule

$$\frac{1}{2} = <J_{tot}> = <S_q> + <L_q> + <S_g>. \tag{27}$$

So, we take $\alpha_m = 0.571$, $\alpha_{ax} = 0.635(0.584)$ and Ψ_g^{E1} in Eq. (24) corresponding to the following spin-coupling-sequence: $\frac{1}{2}(J_{tot}) \to 1_g^- \times \frac{3}{2}(J_q) \to 1_g^- \times \frac{1}{2}(S_q) \times 1(L_q)$. The Ψ_g^{E1}, Eq. (24), was found to give the maximal absolute values of G_A/G_V. The Ψ_g^{M1}-function, Eq. (23), was considered earlier in Ref. [13], but there all higher orbital configurations were neglected. This wave function gives smaller values of G_A/G_V, especially for $\alpha_{ax} = 0.635$, therefore it looks less attractive. Putting, for the simplicity of presentation, $A_3 = 0$ we get the following set of numerical values (those in parentheses correspond to $\alpha_{ax} = 0.584$):

A_g^2	A_0^2	A_1^2	A_2^2	G_A/G_V	$<S_q>$	$<L_q>$
0	0.56(0.74)	0.14(0.18)	0.30(0.075)	0.88(1.27)	0.203(0.42)	0.3(0.08)
0.2	0.52(0.72)	0.05(0.12)	0.22(0.005)	1.0(1.38)	0.23(0.46)	0.336(0.094)

The use of $A_3^2 = 0.2$ helps us to diminish A_1^2 but leaves A_2^2 as big as earlier, or even bigger, thus looking quite inattractive. The most popular value of $\alpha_{ax} = 0.635$ creates the problem of very big A_2^2 (the D-wave probability) and too small G_A/G_V, because inclusion of the renormalization effects will make it, most probably, even smaller. So, the careful re-examination of the hyperon beta-decay theory would be very desirable.

5. Discussion and Conclusion

Our most surprising and most stable result is that despite the validity of the celebrated $SU(6)$-ratio $\mu(P)/\mu(N) = -3/2$, Ref. [14], the $SU(6)$-symmetry is strongly broken, and the mean value of the 56-plet amplitude in the nucleon is considerably less than unity ($A_0^2(N) = 0.55$ to 0.73, depending on the used α_{ax}). The large probability of the $\{70\}_M$, $L_q = 0$, configuration is matching favourably with the idea of diquarks inside nucleons. We have mentioned the apparent difficulty of the very large D-wave in nucleons connected with the use of $\alpha_{ax} = 0.635$ (the broken $SU(3)$ fit). With $\alpha_{ax} = 0.584$ and $A_g^2 \cong 0.2$ we come to acceptably small A_2^2, but this option enforces us to acknowledge the existence of the dominant-hybrid nucleon resonance with $J^P = 1/2^+$ at not-so-large mass. The obvious candidate is the Roper resonance (or resonances) around $M_R \cong 1450$ MeV. The careful investigation of this region in the photo- and electro-excitation experiments is very important. Now, what could be the dynamical origin of the discussed peculiarities? An interesting hint for the answer may come from the solution of the old problem, namely, the large deviation of $\mu(\Delta^+P)_{exp} = 3.0 \div 3.3$ n.m. [15] from $\mu(\Delta^+P)_{SU(6)} = \frac{2\sqrt{2}}{3}\mu(P)$. With the explicit expression of Eq. (19) in mind, we present $\mu(\Delta^+P)$ in the form

$$\mu(\Delta^+P) = \frac{A_0(\Delta)}{A_0(N)^2}\,\mu(\Delta^+P;\{56\}) + \delta\mu(\Delta^+P), \tag{28}$$

where $A_0(\Delta)$ and $A_0(N)$ are the 56-plet amplitudes in the Δ_{33}-resonance and nucleon wave functions, $\delta\mu(\Delta^+P)$ is an unknown contribution due to the higher Δ- and N-orbital configuration. Now, with $A_0(\Delta) \to 1$ (hence, all other $A_i(\Delta) \to 0$ and $\delta\mu(\Delta^+P) \to 0$) and $A_0^2(N) \cong 0.6 \div 0.7$ we remove easily the above-mentioned discrepancy. This situation is realized with the strong $SU(6)$-breaking interactions which are effective in nucleons and weak in Δ's. It is just the instanton-induced qq-interaction [16] which possesses the required properties. Of course, much remains to be done to elaborate and thoroughly check this intriguing possibility.

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